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Effect of the stiffness and inertia of a rib reinforcement on localized bending waves in semi-infinite strips

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ABSTRACT

This paper reports an analytical and numerical study of localized bending waves in a thin elastic isotropic semi-infinite strip with a rib reinforment. Such waves can be considered as spatially non-uniform bending perturbations localized near free edges, similar to Rayleigh waves decaying exponentially with the distance. From the analysis of localized bending waves in thin elastic structures it may be possible to infer the presence of imperfections, cracks or inclusions, in the structure. In this study, a semi-infinite strip, simply supported on two edges and reinforced on the free one, is considered. A general solution is presented and the conditions under which localized bending waves exist are derived. The mathematical model, accompanied by numerical simulations, reveals that the presence of a reinforcement rib can suppress localized bending waves. This effect is mainly due to the stiffness coming from the rib, rather than from its added inertia terms.

1. Introduction

The problem of existence and characterization of localized bending waves in an elastic isotropic semi-infinite plate was first presented by Konenkov (1960). In the West, the first publications on the subject were authored by Sinha (1974) and Thurston and McKenna (1974), that rediscovered the same phenomenon concurrently and independently, without being aware of Konenkov's contribution. Such a solution has also been represented recently by Kauffmann (1998). Norris (1994) presented the results for flexural edge waves in orthotropic thin plates. In the same year, Ambartsumyan and Belubekyan (1994) considered localized bending waves along the edge of a plate using several nonclassical plate theories, concluding that Timoshenko–Mindlin plates do not admit localized edge waves. Norris et al. (2000) showed, on the other hand, that such flexural edge waves do exist when Mindlin's plate theory is considered.

These localized waves are also called "edge modes". The edge resonance phenomenon in circular isotropic disks has been discovered experimentally by Shaw (1956), when analyzing the modes of thick piezoelectric disks. Bobrovnitskii and Korotkov (1991) noted that these local resonances exist virtually in all engineering structures. Their work provides detailed analytical results for several infinitely long homogeneous structures perturbated by reactive elements. The specific case of edge resonances in plates has been tackled by Auld and Tsao (1977), Grinchenko and Meleshko, 1975 and, more recently, by Le Clezio et al. (2003).

One of the latest developments in the field has been the extension to general anisotropic media, by Zakharov and Becker (2003). The above list of contributions in the field of localized bending waves and edge modes is not to be considered exhaustive, and more works (in particular, experimental results) can be found in the references of the cited publications. A possible application of localized bending waves is the detection of imperfections, cracks or inclusions, in thin elastic structures.

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Nomenclature

a, b	plate dimensions in the x and y directions, respectively
a_0, b_0	dimensions of rectangular cross-section reinforcement rib, a_0 parallel to z
a_{max}, a_{min}	$a_{max} = \max(a_0, b_0), a_{min} = \min(a_0, b_0)$
d	diameter of circular cross-section reinforcement rib
D	plate flexural rigidity, $D = 2Eh^3/3(1-\nu^2)$
E	Young modulus of the strip material
E_0	Young modulus of the rib material
$f_n(x)$	eigenfunction in the x direction
G_0	shear modulus of the rib material, $G_0 = E_0/2(1+\nu_0)$
$2h$	plate thickness
I_p	rib cross-sectional polar moment of inertia
I_t	rib cross-sectional torsional moment of inertia
J	rib cross-sectional bending moment of inertia
$K(\eta_n)$	$K(\eta_n) = 0$ is the condition for existence of a bending wave of frequency η_n, as in Eq. (11)
n	mode number
p_1, p_2	functions of η_n, as in Eq. (9)
S	rib cross-sectional area
w	plate mid-surface normal displacement
x, y, z	coordinates along the two sides and the thickness of the strip
β_1, β_2	dimensionless parameters, defined in Eqs. (12)
β_3	dimensionless parameter, defined in Eqs. (23)
γ_1, γ_2	dimensionless parameters, defined in Eqs. (12)
γ_3	dimensionless parameter, defined in Eqs. (23)
η_n	dimensionless frequency, see Eq. (7)
η^*	dimensionless frequency for all localized bending waves for a strip without reinforcement, as in Eq. (17)
λ_n	wave number for the eigenfunction, $\sin(\lambda_n y)$, in the y direction, $\lambda_n = n\pi/b$
ν	Poisson ratio of the strip material
ν_0	Poisson ratio of the rib material
ω_n	natural frequency of mode n
ρ	bulk density of the strip material
ρ_0	bulk density of the rib material
Δ^2	bi-harmonic operator, $\Delta^2 = \partial^4/\partial x^4 + 2\partial^4/(\partial x^2 \partial y^2) + \partial^4/\partial y^4$

In a recent work, Belubekyan et al. (2007) considered the existence and properties of localized bending waves in an orthotropic plate with a rib reinforcement. The semi-infinite plate of finite width was simply supported along two edges, with a rigid rib at the free edge.

This work considers the same geometry for the plate and the reinforcement of Belubekyan et al. (2007), though for the reinforcement both the stiffness and the inertia terms, not present in Belubekyan et al. (2007), are included. Analyses and parametric investigations are reported, providing a better understanding of the phenomenon of localized bending waves with such a geometry. If the semi-infinite strip provides the homogeneous structure for such waves, the inhomogeneity that makes the localized modes possible is represented by both the edge and the rib.

The rest of the paper is organized as follows. In Section 2, the mathematical model is presented, starting from the differential equations of motion. The solution of interest is found; correspondingly, the necessary and sufficient conditions for the existence of localized bending waves are derived. The limiting cases obtained neglecting either the inertia or the stiffness terms of the rib are presented. Results are then reported in Section 3 for circular and rectangular cross-sections of the rib. Changes in the dimensionless frequencies of the first localized modes, as well as regions of existence of such modes, are studied through a series of numerical investigations. Pertinent conclusions are then reported.

2. Mathematical modeling

A semi-infinite strip with two simply supported edges and one rib-reinforced edge, as sketched in Fig. 1, is considered. The width of the strip is b and the thickness $2h$. The cartesian reference system (x, y, z) is chosen so that the plane (xOy)

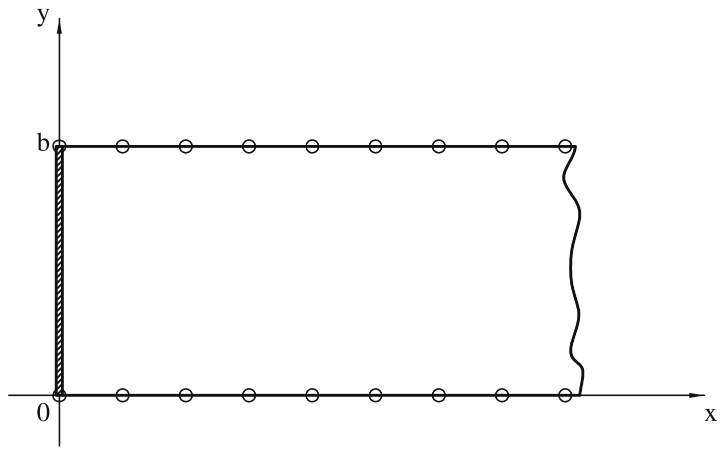

Fig. 1. Sketch of semi-infinite strip.

is coincident with the strip middle surface, while z is the coordinate along the thickness; the axes x and y are aligned along the edges.

The governing equation for the bending vibrations of an isotropic plate, based on Kirchhoff thin-plate theory (Timoshenko and Woinowsky-Krieger, 1959), can be written as

$$D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where $w(x, y, t)$ is the plate mid-surface normal displacement, ρ is the bulk density of the plate material and D represents the plate flexural stiffness. For a plate of thickness $2h$, the latter can be expressed as $D = 2Eh^3/3(1-\nu^2)$, where E and ν are the Young modulus and Poisson ratio of the plate material, respectively. The Δ operator is the two-dimensional Laplacian. Such a model for the flexural vibrations of a thin plate is also known as Germain–Lagrange model (Bobrovnitskii and Korotkov, 1991).

For the simply supported edges, the boundary conditions are

$$\begin{cases} w = 0 \\ \dfrac{\partial^2 w}{\partial y^2} = 0 \end{cases} \quad y = 0, \quad y = b \tag{2}$$

The edge $x = 0$ is free (there are no bending, twisting or shear forces applied) and reinforced with a rib, that can be modeled by an elastic supporting beam. In the static case, the boundary conditions at the reinforced edge can be found in the book by Timoshenko and Woinowsky-Krieger (1959). Taking into account the inertia of the beam, the resulting boundary conditions at $x = 0$, are written as

$$\begin{cases} D\left[\dfrac{\partial^2 w}{\partial x^2} + \nu \dfrac{\partial^2 w}{\partial y^2}\right] + G_0 I_t \dfrac{\partial^3 w}{\partial x \partial y^2} - \rho_0 I_p \dfrac{\partial^3 w}{\partial x \partial t^2} = 0 \\ D\left[\dfrac{\partial^3 w}{\partial x^3} + (2-\nu)\dfrac{\partial^3 w}{\partial x \partial y^2}\right] + E_0 J \dfrac{\partial^4 w}{\partial y^4} + \rho_0 S \dfrac{\partial^2 w}{\partial t^2} = 0 \end{cases} \quad x = 0 \tag{3}$$

Here, $G_0 = E_0/2(1+\nu_0)$ is the shear modulus of the beam material, while ρ_0, E_0 and ν_0 are its bulk density, Young modulus and Poisson ratio, respectively. S is the beam cross-sectional area, I_t its torsional moment of inertia, I_p its polar moment of inertia and J its bending moment of inertia.

One additional boundary condition is needed. If the strip is semi-infinite, the localization condition prescribes an attenuation as $x \to \infty$, hence an additional constraint is

$$\lim_{x\to\infty} w = 0 \tag{4}$$

Solutions of Eq. (1) satisfying the boundary conditions of Eqs. (2)–(4) represent – if they exist – localized bending waves near the edge $x = 0$. If such solutions are assumed in the form

$$w_n(x, y, t) = f_n(x) e^{\iota \omega_n t} \sin(\lambda_n y) \tag{5}$$

then the boundary conditions of Eqs. (2) are automatically satisfied, when taking $\lambda_n = n\pi/b$. The function $f_n(x)$ can then be determined by solving the ordinary differential equation

$$\frac{d^4 f_n}{dx^4} - 2\lambda_n^2 \frac{d^2 f_n}{dx^2} + \lambda_n^4 (1 - \eta_n^2) f_n = 0 \tag{6}$$

where the parameter η_n is a dimensionless frequency, defined as

$$\eta_n^2 = \frac{2\rho h \omega_n^2}{\lambda_n^4 D} \tag{7}$$

The attenuation condition of Eq. (4) implies that $f_n(x) \to 0$ as $x \to \infty$. Therefore, the general solution of Eq. (6) is in the form

$$f_n(x) = C_{1n} e^{-\lambda_n p_1 x} + C_{2n} e^{-\lambda_n p_2 x} \tag{8}$$

where p_1 and p_2 are given by

$$p_1 = \sqrt{1 + \eta_n}, \quad p_2 = \sqrt{1 - \eta_n} \tag{9}$$

The two roots p_1 and p_2 correspond to a fast and a slow decaying mode, in the x direction. As η_n approaches the limiting value 1, the wave of order n clearly becomes less and less localized. From Eqs. (9), the dimensionless frequency η_n ranges from 0 to 1, that is

$$0 < \eta_n < 1 \tag{10}$$

The ratio of the constants C_{1n} and C_{2n} can be obtained imposing the boundary conditions at the reinforced edge, Eqs. (3), leading to a linear homogeneous system in C_{1n} and C_{2n}. The nontrivial solution is given by posing the determinant of the matrix of the coefficients to zero. That yields the following equation in η_n:

$$K(\eta_n) = p_1^2 p_2^2 + 2(1 - \nu) p_1 p_2 - \nu^2 + \beta_1 \gamma_1 (1 - \beta_2 \eta_n^2)(1 - \gamma_2 \eta_n^2) + [\gamma_1 (1 - \gamma_2 \eta_n^2) + \beta_1 (1 - \beta_2 \eta_n^2) p_1 p_2](p_1 + p_2) = 0 \tag{11}$$

where $\beta_1, \beta_2, \gamma_1$ and γ_2 are dimensionless quantities, defined as:

$$\beta_1 = \frac{G_0 I_t \lambda_n}{D}, \quad \beta_2 = \frac{\lambda_n^2 D \rho_0 I_p}{2\rho h G_0 I_t}, \quad \gamma_1 = \frac{E_0 J \lambda_n}{D}, \quad \gamma_2 = \frac{\rho_0 S D}{2\rho h E_0 J} \tag{12}$$

The function $K(\eta_n)$ is continuous and monotonically decreasing in $\eta_n \in (0, 1)$. The monotonicity can be verified without recurring to differentiation, since all the individual addenda (or their factors) in Eq. (11) that are function of η_n decrease monotonically with η_n in the interval of interest. Evaluating $K(\eta_n)$ at the limits $\eta_n = 0$ and $\eta_n = 1$ yields

$$K(0) = 1 + 2(1 - \nu) - \nu^2 + \beta_1 \gamma_1 + 2(\gamma_1 + \beta_1) \tag{13}$$

and

$$K(1) = -\nu^2 + \sqrt{2}\gamma_1 (1 - \gamma_2) + \beta_1 \gamma_1 (1 - \beta_2)(1 - \gamma_2) \tag{14}$$

For the so-called natural values of ν, i.e. for $0 < \nu < 1/2$, $K(0) > 0$, while the sign of $K(1)$ depends on the geometrical, inertial and elastic properties of the strip and of the reinforcing rib.

For the case without rib, Eq. (11) becomes

$$K(\eta_n) = p_1^2 p_2^2 + 2(1 - \nu) p_1 p_2 - \nu^2 = 0 \tag{15}$$

In this case, the values of $K(\eta_n)$ at the boundaries $\eta_n = 0$ and $\eta_n = 1$ are

$$K(0) = 3 - 2\nu - \nu^2 > 0, \quad K(1) = -\nu^2 < 0 \tag{16}$$

Hence, there is one η^* in $(0, 1)$ for which $K(\eta^*) = 0$. In a straightforward way, this η^* can be derived from Eq. (15) as

$$\eta^* = \sqrt{1 - (\nu - 1 + \sqrt{1 - 2\nu + 2\nu^2})^2} \tag{17}$$

This case is similar to the standing Konenkov's wave in a semi-infinite thin plate. An expression analogous to Eq. (17) can be found in Konenkov (1960).

When no reinforcement is present, infinite localized bending waves exist, all sharing the same value of dimensionless frequency η^*, that depends only on the Poisson ratio ν. For the range $0 < \nu < 1/2$, η^* is very close to 1; for example, considering a typical aluminum alloy, $\nu = 0.34$, that corresponds to $\eta^* = 0.9966$.

The presence of a reinforcement rib can make $K(1)$ positive. In that case – from the continuity and monotonicity of $K(\eta_n)$ – there are no solutions of Eq. (11). Physically, this implies that the localized bending wave, for a given n, disappears. The borderline condition between existence and non-existence of the wave can be found imposing $K(1) = 0$. In general, the introduction of a reinforcement rib at the edge of the strip eliminates only certain localized bending waves, usually the high frequencies ones (as reported later in Section 3). Eqs. (12) contain a dependence on n through λ_n, as $\lambda_n = n\pi/b$. Therefore, the sign of $K(1)$ as given in Eq. (14) depends on n and the existence of the localized wave itself depends on n. This implies that such an analysis should be performed for different waves, corresponding to different n.

Eq. (14) may erroneously induce to think that when $\gamma_2 = 1$ the reinforcement on the free edge has no effect on the localized waves, since $K(1)$ becomes $-\nu^2$, as in the case without any rib. This condition guarantees the existence of the wave (of any order n), though the corresponding dimensionless frequency η_n is in general different from the η^* of Eq. (17) (dimensionless frequency when no reinforcement is present). When $\gamma_2 = 1$, the contribution of a reinforcement rib does still appear in Eq. (11), and determines the value of η_n.

To conclude the mathematical modeling, the equations for two limiting cases are presented, when either the inertial or the elastic terms of the rib are neglected. If the inertia of the reinforcement rib is not considered, $K(\eta_n)$ simplifies to

$$K(\eta_n) = p_1^2 p_2^2 + 2(1-\nu)p_1p_2 - \nu^2 + \beta_1\gamma_1 + (\gamma_1 + \beta_1 p_1 p_2)(p_1 + p_2) = 0 \tag{18}$$

At the boundaries $\eta_n = 0$ and $\eta_n = 1$, $K(\eta_n)$ assumes the values

$$K(0) = 1 + 2(1-\nu) - \nu^2 + \beta_1\gamma_1 + 2(\gamma_1 + \beta_1) \tag{19}$$

and

$$K(1) = -\nu^2 + \gamma_1\left(\sqrt{2} + \beta_1\right) = -\nu^2 + \frac{E_0 J \lambda_n}{D}\left(\sqrt{2} + \frac{G_0 I_t \lambda_n}{D}\right) \tag{20}$$

The mass properties of the strip, represented by ρ, do not appear in Eq. (20). This implies that the existence of localized bending waves is determined by the relative weight of only stiffness terms, appearing with a positive sign in Eq. (20). Therefore, the bending and torsional stiffnesses of the rib work together towards the elimination of bending waves in the strip. Eq. (20) is equivalent to the one derived by Belubekyan et al. (2007).

On the other hand, when the stiffness terms of the rib are assumed to be zero or negligible and only its inertia is considered, the boundary conditions of Eq. (3) need to be modified as

$$\begin{cases} D\left[\dfrac{\partial^2 w}{\partial x^2} + \nu\dfrac{\partial^2 w}{\partial y^2}\right] - \rho_0 I_p \dfrac{\partial^3 w}{\partial x \partial t^2} = 0 \\ D\left[\dfrac{\partial^3 w}{\partial x^3} + (2-\nu)\dfrac{\partial^3 w}{\partial x \partial y^2}\right] + \rho_0 S \dfrac{\partial^2 w}{\partial t^2} = 0 \end{cases} \quad x = 0 \tag{21}$$

Accordingly, $K(\eta_n)$ can be derived as

$$K(\eta_n) = p_1^2 p_2^2 + 2(1-\nu)p_1p_2 - \nu^2 + \beta_3\gamma_3\eta_n^4 - \eta_n^2(\gamma_3 + \beta_3 p_1 p_2)(p_1 + p_2) = 0 \tag{22}$$

having defined the following dimensionless quantities

$$\beta_3 = \frac{\rho_0 I_p \lambda_n^3}{2\rho h}, \quad \gamma_3 = \frac{\rho_0 S \lambda_n}{2\rho h} \tag{23}$$

Then, evaluating $K(\eta_n)$ at the limits $\eta_n = 0$ and $\eta_n = 1$ returns

$$K(0) = 1 + 2(1-\nu) - \nu^2 + \beta_1\gamma_1 + 2(\gamma_1 + \beta_1) \tag{24}$$

and

$$K(1) = -\nu^2 + \gamma_3(\beta_3 + \sqrt{2}) = -\nu^2 + \frac{\lambda_n \rho_0 S}{2\rho h}\left(\frac{\lambda_n^3 \rho_0 I_p}{2\rho h} - \sqrt{2}\right) \tag{25}$$

In this case, the flexural stiffness of the strip D has no influence and the existence of the localized bending waves is given by the relative weight of inertia terms only. The inertia of the rib $\rho_0 I_p$ appears with a positive sign, while the cross-sectional mass $\rho_0 S$ has a negative sign. Therefore, one of the inertia terms of the rib $(\rho_0 S)$ contributes to the presence of localized waves in the strip, while the other $(\rho_0 I_0)$ promotes their suppression.

The case when no rib at all is present, i.e. Eq. (15), can be derived from either Eq. (18), in the limit $\beta_1 = \gamma_1 = 0$, or from Eq. (22), taking $\beta_3 = \gamma_3 = 0$.

3. Results and discussion

The mathematical development reported in Section 2 is valid for different geometries of the rib reinforcement; furthermore, it allows the materials of the strip and of the rib to be different. In this section, in order to infer the effects of such a rib on the localized bending waves, results are reported for more specific cases. In all the following, the material of the strip and of the rib is assumed to be the same. Values for a typical aluminum alloy are used, $E = E_0 = 70$ GPa, $\nu = \nu_0 = 0.34$ and $\rho = \rho_0 = 2700$ kg/m^3. The value $b = 1$ m is taken for the width of the strip. Circular and rectangular cross-sections are considered for the rib. The properties of the circular cross-section of diameter d are computed as follows:

$$S = \frac{\pi d^4}{4}, \quad J = \frac{\pi d^4}{64}, \quad I_t = \frac{\pi d^4}{32}, \quad I_p = \frac{\pi d^4}{32} \tag{26}$$

For the rectangular cross-section of sides a_0 and b_0 (with a_0 parallel to the z axis of the strip), the following formulae are used (Timoshenko and Goodier, 1969):

$$S = a_0 b_0, \quad J = \frac{b_0 a_0^3}{12}, \quad I_p = \frac{a_0 b_0 (a_0^2 + b_0^2)}{2} \tag{27}$$

For the torsional constant I_t, the following series expansion is used (Timoshenko and Goodier, 1969)

$$I_t = \frac{a_{max} a_{min}^3}{3} \left[1 - \frac{192}{\pi^5} \frac{a_{min}}{a_{max}} \sum_{n=1,3,5,\cdots}^{\infty} \frac{1}{n^5} \tanh \frac{n\pi a_{max}}{2a_{min}} \right] \tag{28}$$

where $a_{min} = \min(a_0, b_0)$ and $a_{max} = \max(a_0, b_0)$. Both the circular and the rectangular reinforcements are positioned centered at the strip free edge.

The circular and square cross-sections cases are well suited for parametric analyses, since a single parameter – the diameter for the circular case, the side length for the square one – controls all the inertial and elastic properties of the rib. Hence, the dimensionless quantities $\beta_1, \beta_2, \gamma_1$ and γ_2 (or β_3 and γ_3), that characterize the bending waves (defined in Section 2), depend on a single parameter. The limiting case when the area S of the reinforcement rib goes to 0 is included in the parametric analyses to consider the base case without reinforcement. Figs. 2 and 3 report the dimensionless frequencies η_n (for $n = 1, 2$

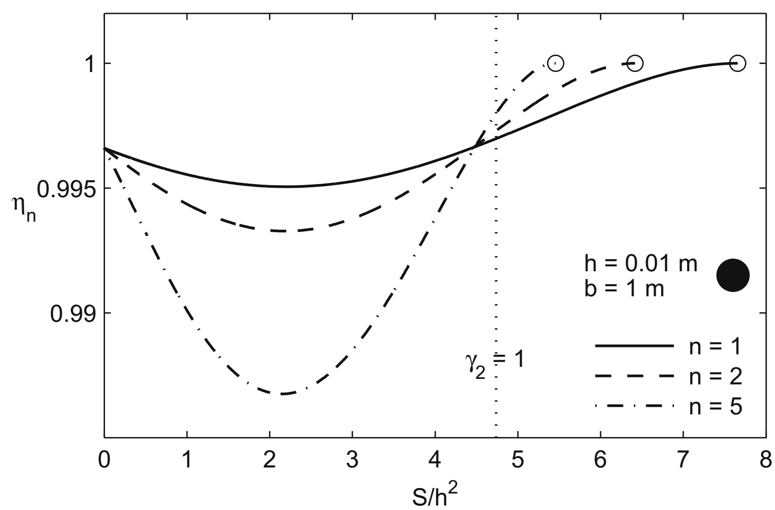

Fig. 2. Dimensionless frequencies of localized bending waves for circular cross-sections.

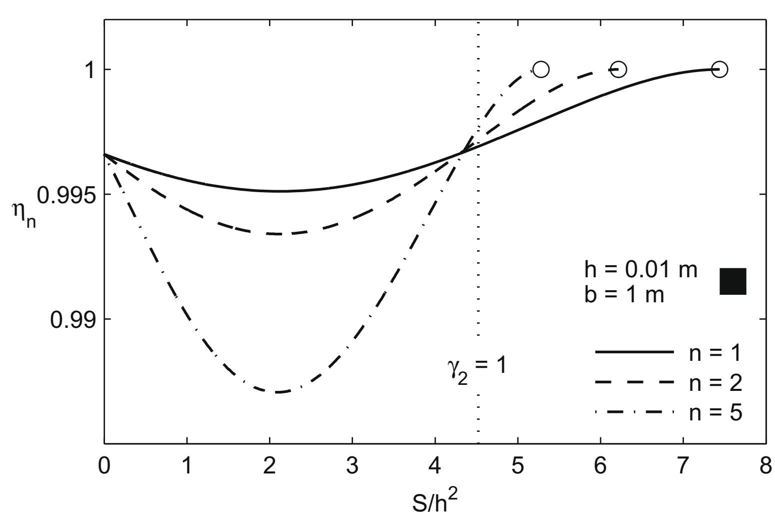

Fig. 3. Dimensionless frequencies of localized bending waves for square cross-sections.

and 5) of the localized bending waves in the strip as a function of the area S of a circular and square rib, respectively. The thickness of the strip is taken as $2h = 0.02$ m. In this case, the two geometries for the cross-section of the reinforcement yield very similar results. When no rib is present ($S/h^2 = 0$), the curves for the different waves start all at the same η^*, function only of the Poisson ratio ν of the strip material, as given in Eq. (17). As a rib reinforcement is introduced, initially the dimensionless frequencies η_n decrease slightly. Then, they start to increase, till they reach the unit value. This marks the suppression of the corresponding localized bending wave, signaled by circles in the plots. The variation of η_n is anyway limited around a neighborhood of η^*. From these results, it seems that there exists a particular dimension of the cross-section – both in the circular and square case – for which the normalized frequencies η_n for the various modes coincide, and they are equal to η^*. This peculiar intersection of the curves is found to be close to the dimension of the rib that corresponds to $\gamma_2 = 1$. This value can be found from Eq. (12) and it is equal to

$$(S/h^2)_{\gamma_2=1} = \frac{4\pi}{3(1-\nu^2)} \tag{29}$$

for the circular section, and to

$$(S/h^2)_{\gamma_2=1} = \frac{4}{(1-\nu^2)} \tag{30}$$

for the square section, having assumed $E = E_0$ and $\rho = \rho_0$. The similar effect of the two different cross-sections observed in Figs. 2 and 3 is confirmed by Eqs. (29) and (30), that yield almost the same value of S/h^2 for $\gamma_2 = 1$.

By varying the thickness of the strip $2h$, it is possible to obtain the regions of existence of the localized bending waves as a function of this parameter. Fig. 4 depicts these regions for the circular cross-section, for $n = 1, 2$ and 5. When no reinforcement is present, that is $S/h^2 = 0$, such waves exist for any n. If the rib is present, for a given thickness of the strip there exists a critical dimension that eliminates the localized wave of order n. A larger reinforcement is needed to eliminate lower frequency waves. Conversely, as the strip itself becomes more rigid (h/b increases) less reinforcement material is needed to make the localized waves disappear. For particular combinations of strip and rib stiffnesses, there is a certain number of low frequency waves that are present, while higher frequency ones disappear. The effect of the strip width b can also be observed. For a given thickness $2h$, increasing b corresponds to moving towards the left in the plot, so wider reinforcements are needed to suppress localized bending waves.

The relative role played by the elastic and the inertial characteristics of the reinforcement rib is now assessed. The analysis is performed with a circular cross-section. Fig. 5 reports the case of a massless rib, obtained using Eq. (20). In general, if the mass of the rib is omitted, a slightly smaller section is needed to eliminate the localized waves. The results are very similar to the one of Fig. 4, thus indicating that – for this configuration – the stiffness terms of the rib are predominant over the inertial ones for the determination of localized waves in the strip. This is confirmed by the dual case, i.e. that of an infinitely flexible rib with only inertial terms, analyzed in Fig. 6. The results are still qualitatively the same as those in Figs. 4 and 5, though much larger values of S/h^2 are necessary to make the localized waves disappear. This is in accordance with Eq. (25), that predicts that one of the inertia terms of the rib contributes to the presence of the waves, while the other promotes their suppression.

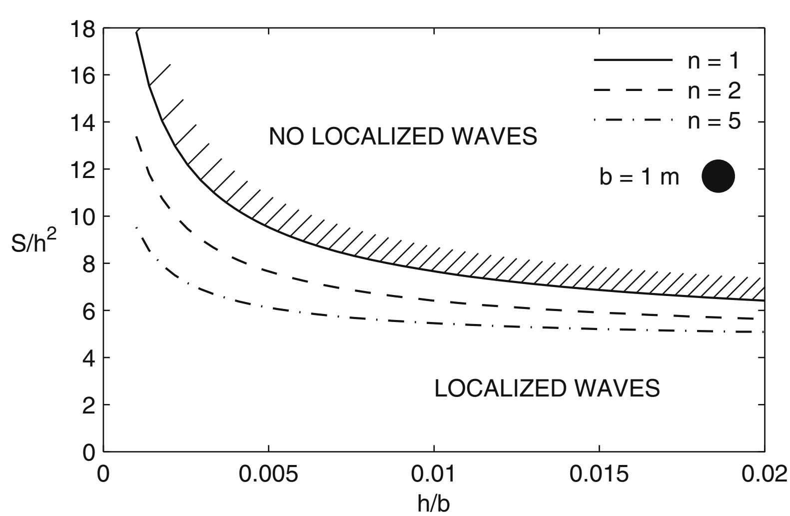

Fig. 4. Regions of existence of localized bending waves, circular cross-sections.

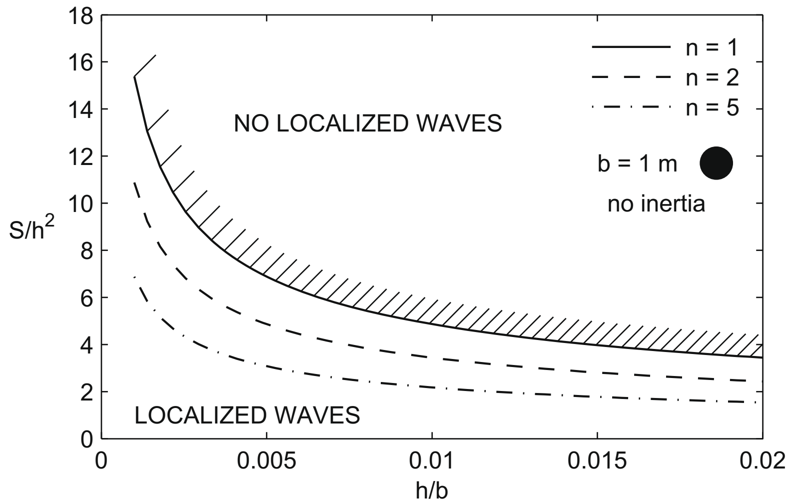

Fig. 5. Regions of existence of localized bending waves, circular cross-sections without inertia.

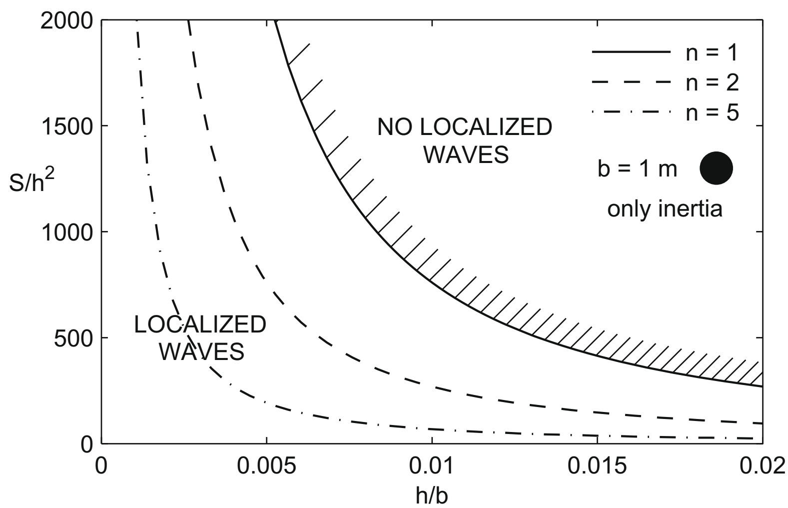

Fig. 6. Regions of existence of localized bending waves, circular cross-sections with only inertia.

The case of a square cross-section is analyzed in terms of regions of existence of localized waves in Fig. 7, where both stiffness and inertia terms are included. The regions are almost coincident with the ones found for the circular cross-section, reported in Fig. 4. This further witnesses the similarity of these two geometries for this problem.

Finally, the case of rectangular cross-sections is analyzed in Fig. 8, where the thickness of the strip is held constant, as $2h = 0.02$ m . The sides a_0 and b_0 are oriented with a_0 parallel to the z axis of the strip. A monotonic trend is found, with reinforcements that are more and more effective in the elimination of localized waves as the ratio a_0/b_0 increases. In fact, when a_0/b_0 is larger, a smaller dimensionless cross-section S/h^2 is needed to suppress a wave of given n. This can be explained by the fact that, for the same area S, such an arrangement of material produces a larger stiffness in the proper direction (see, for example, the bending moment of inertia J), that opposes to a bending wave.

4. Conclusions

The problem of localized bending waves in an elastic semi-infinite strip with a rib reinforcement has been analyzed. The mathematical conditions for the existence of the waves have been derived from the equation of motion. In particular, the effects of inertial and elastic terms in the rib have been separately investigated, leading to an interesting duality. With such a configuration, the existence of localized bending waves for a massless rib reinforcement does not depend on the inertial properties of the strip. On the other hand, if only the inertia contributions of the rib are taken into account, the flexural stiffness of the strip plays no role. Results for several cross-sections and a typical aluminum alloy material have been presented.

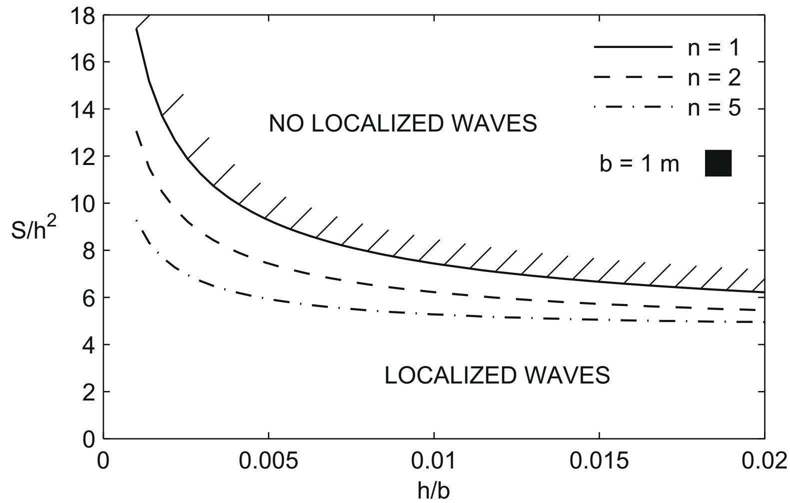

Fig. 7. Regions of existence of localized bending waves, square cross-sections.

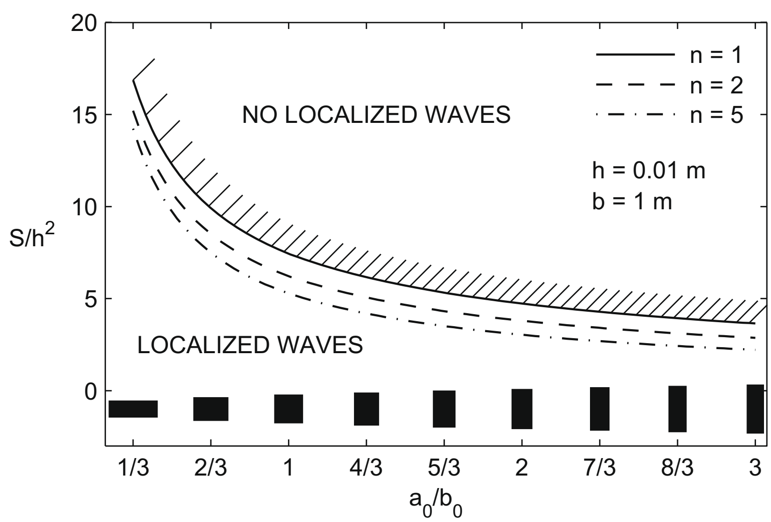

Fig. 8. Regions of existence of localized bending waves, rectangular cross-sections.

Analyzing the changes in the dimensionless frequencies of the waves, it has been found that – for a circular and square cross-sections – there exists a particular dimension for which the edge waves are equivalent to the edge waves occurring without any reinforcement. Investigating the regions of existence of the waves themselves, it appeared that the elastic terms of the rib are predominant over the inertial ones, for this problem. Furthermore, stiffer strips have been found to require smaller reinforcements to suppress their edge waves.

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