

Review Notes for Week 6

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During this week, we mainly focused on two topics (i) strain-displacement relations and transformation of strain components from one co-ordinate system to another using Mohr circle, and (ii) Hooke's law for 2-D and three-dimensional states of stress.

1 Lecture # 1, October 1,2001

We discussed strain-displacement relations. Let u , v , and w are the components of the displacement, at a point, in the x , y , and z -directions respectively. As was the case for the stress, strain is also a point function. That is it varies from a point to point. The strain at a point is expressed in terms of normal and shear strain components. The strain components used in this course are the so-called infinitesimal strains, *i.e.* the components are far far less than one. Typically they are of the order of 10^{-4} . To understand the concept of normal strain in any direction, say α , consider a fiber AB of vanishingly small length, Δs , lying along the given direction, before the

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structure is subjected to given loads. After deformation, assume that the fiber length has become $\Delta s'$. The normal strain, ϵ_α , along the direction α , is given as:

$$\epsilon_\alpha = \lim_{\Delta s \rightarrow 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

The idea of vanishingly small fiber implies that a given strain component can vary from point to point.

The three normal strain components are: ϵ_x , the strain component in the x -direction, ϵ_y , the strain component in the y -direction, and ϵ_z , the strain component in the z -direction.

These components are given as:

$$\begin{aligned}\epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_y &= \frac{\partial v}{\partial y} \\ \epsilon_z &= \frac{\partial w}{\partial z}\end{aligned}\tag{1}$$

The complete derivation of these normal strain-displacement relations is given on page 16-18 of your text.

To understand the concept of shear strain, consider two fibers which are orthogonal to each other, *i.e.* make an angle of $\pi/2$ to each other. Then the infinitesimal shear strain is the change in the angle from $\pi/2$. The three infinitesimal shear strain components are given as:

$$\begin{aligned}\gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \gamma_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\end{aligned}\tag{2}$$

The derivation of the above equations is given on page 18 to 19 of your text.

2 Lecture # 2, October 3, 2001

This lecture was mostly on the transformation of strain components given in one co-ordinate system to those in another co-ordinate system for the case of plane strain conditions.

Plane strain conditions are used to analyze those structures for which the strain components, ϵ_{zz} , γ_{xz} , and γ_{yz} are zero.

Let ϵ_x , ϵ_y and γ_{xy} are the three non-zero stress components for a plane strain condition. Consider another co-ordinate system $x' - y'$ such that x' -axis makes an angle θ (considered positive if in anti-clockwise direction) with the x -axis. Let $\epsilon_{x'}$, $\epsilon_{y'}$, and $\gamma_{x'y'}$ are the strain components in the new co-ordinate system. These components are given as:

$$\begin{aligned}\epsilon_{x'} &= \epsilon_x \cos^2 \theta + \epsilon_y \sin^2 \theta + \gamma_{xy} \cos \theta \sin \theta \\ \epsilon_{y'} &= \epsilon_x \sin^2 \theta + \epsilon_y \cos^2 \theta - \gamma_{xy} \cos \theta \sin \theta \\ \gamma_{x'y'} &= (\epsilon_x - \epsilon_y) \sin 2\theta - \gamma_{xy} \cos 2\theta\end{aligned}\tag{3}$$

Derivation of Eq. 3 is given on page 21-23 of your text. The direction for the principal strain can be obtained by letting $\gamma_{x'y'}$ approach zero in the third of the above equations and solving for θ . If θ_p is the direction of the principal axis, it is given as:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

Note that Eq. 3 are similar to those used for stress transformation and thus, given $\epsilon_{x'}$, $\epsilon_{y'}$, and $\gamma_{x'y'}$, the strain along an any direction can be obtained using the Mohr's circle plotted using $\gamma/2$ vs. ϵ . The text by Beer and Johnston provides an excellent description of Mohr circle for strain.

One of the application of Eq. 3 is in determining the shear strain at a point. To that end, we place three strain gages to measure the strain in three different directions at that point. Assume that one of the strain gage is placed along the x -direction, one along the y -direction (i.e. normal to the x -direction) and the third along the x' -direction making an angle θ with the x -direction. Then using first of the transformation equation, we can obtain the value of γ_{xy} .

3 Lecture # 3, October 5, 2001

In this lecture, we talked about Hooke's law for linear elastic materials in three-dimensions. An isotropic material can be expressed using two material constants E , the Young's modulus and ν , the Poisson's ratio. A material is called isotropic, if the material properties are independent of the direction. When an isotropic material is pulled, by a direct stress, say, σ_x , it only elongates and does not possess any shear strain. The resulting strain ϵ_x , in the x -direction is given as:

$$\epsilon_x = \frac{\sigma_x}{E}$$

The application of the direct stress σ_x also results in transverse strains ϵ_y and ϵ_z given as;

$$\begin{aligned}\epsilon_y &= -\nu\epsilon_x = -\nu\frac{\sigma_x}{E} \\ \epsilon_z &= -\nu\epsilon_x = -\nu\frac{\sigma_x}{E}\end{aligned}$$

The value of the Poisson's ratio can lie between 0 and 0.5, but for most of the metals it generally lies between 0.2 and 0.33. The materials with Poisson's ratio equal to 0.5 are called incompressible media, an example being rubber. For a material point, subjected to stresses, σ_x , σ_y , and σ_z , the normal strain components, ϵ_x , ϵ_y , and ϵ_z are given as:

$$\begin{aligned}\epsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \\ \epsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \\ \epsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_y + \sigma_x)]\end{aligned}$$

For an isotropic material, the shear strain γ_{xy} , for example, is related to the shear stress, τ_{xy} , through the shear modulus, G ; $\tau_{xy} = G\gamma_{xy}$. The shear modulus is not an independent material constant. It is related to the young's modulus and the Poisson's ratio as follows:

$$G = \frac{E}{2(1 + \nu)}$$

Excellent discussion on Hooke's law appears on pp. 85-98 in the text by Beer, Johnston and Dewolfe.