I. Centroid

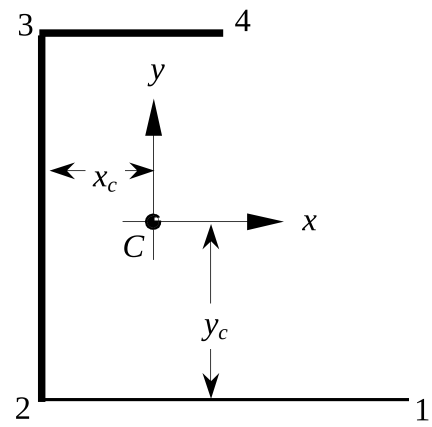

Fig. 4 Calculation of the centroid.

Let's place the origin at point 2 and calculate x_c and y_c.

Section	x_i	y_i	A_i
12	a	0	$(2a)(t)$
23	0	a	$(2t)(2a)$
34	$a/2$	$2a$	$(a)(2t)$

$$
\begin{aligned}
\sum A_i &= 8\,a\,t \\
\sum x_i\,A_i &= 3\,a^2\,t \\
\sum y_i\,A_i &= 8\,a^2\,t
\end{aligned}
$$

$$
\begin{aligned}
x_c &= \frac{\sum x_i\,A_i}{\sum A_i} = \frac{3}{8}\,a \\
y_c &= \frac{\sum y_i\,A_i}{\sum A_i} = a
\end{aligned}
$$

$$\boxed{x_c = \frac{3}{8}\,a \qquad y_c = a} \tag{4}$$ Ans.

II. Second moments of area

Using thin-walled assumption

Section	$I_{x_{ci}}$	$I_{y_{ci}}$	$I_{x_c y_{ci}}$
12	$(2a\,t^3)/12 \approx 0$	$\left(t\,(2a)^3\right)/12$	0
23	$\left(2t\,(2a)^3\right)/12$	$\left(2a\,(2t)^3\right)/12 \approx 0$	0
34	$\left(a\,(2t)^3\right)/12 \approx 0$	$\left(2t\,(a)^3\right)/12$	0

Second Moment of area I_{xx} using thin-walled assumption is

$$\begin{aligned} I_{xx12} &= I_{xc_{12}} + A_{12}\,[dy_{12}]^2 \\ &= 0 + (2a)(t)\,[-a]^2 \end{aligned}$$

$$\begin{aligned} I_{xx23} &= I_{xc_{23}} + A_{23}\,[0]^2 \\ &= \frac{16}{12}\,t\,a^3 \end{aligned}$$

$$\begin{aligned} I_{xx34} &= I_{xc_{34}} + A_{34}\,[dy_{34}]^2 \\ &= 0 + (a)(2t)\,[a]^2 \end{aligned}$$

$$\begin{aligned} I_{xx} &= I_{xx12} + I_{xx23} + I_{xx34} \\ &= 2\,t\,a^3 + \frac{4}{3}\,t\,a^3 + 2\,t\,a^3 = \frac{16}{3}\,t\,a^3 \end{aligned}$$

Second Moment of area I_{yy} using thin-walled assumption is

$$\begin{aligned} I_{yy12} &= I_{y_{c12}} + A_{12}\,[dx_{12}]^2 \\ &= \frac{8}{12}\,t\,a^3 + (2a)(t)\left[\frac{5}{8}\,a\right]^2 \end{aligned}$$

$$\begin{aligned} I_{yy23} &= I_{y_{c23}} + A_{23}\,[dx_{23}]^2 \\ &= 0 + (2a)(2t)\left[-\frac{3}{8}\,a\right]^2 \end{aligned}$$

$$\begin{aligned} I_{yy34} &= I_{y_{c34}} + A_{34}\,[dx_{34}]^2 \\ &= \frac{2}{12}\,t\,a^3 + (a)(2t)\left[\frac{1}{8}\,a\right]^2 \end{aligned}$$

$$\begin{aligned} I_{yy} &= I_{yy12} + I_{yy23} + I_{yy34} \\ &= \left(\frac{8}{12}\,t\,a^3 + \frac{50}{64}\,t\,a^3\right) + \left(0 + \frac{36}{64}\,t\,a^3\right) + \left(\frac{1}{6}\,t\,a^3 + \frac{2}{64}\,t\,a^3\right) = \frac{53}{24}\,t\,a^3 \end{aligned}$$

Second Moment of area I_{xy} using thin-walled assumption is

$$\begin{aligned} I_{xy12} &= I_{x_c\,y_{c12}} + A_{12}\,[dx_{12}\,dy_{12}] \\ &= 0 + (2a)(t)\left[\left(\frac{5}{8}\,a\right)(-a)\right] \end{aligned}$$

$$\begin{aligned} I_{xy23} &= I_{x_c\,y_{c23}} + A_{23}\,[dx_{23}\,dy_{23}] \\ &= 0 + (2a)(2t)\left[\left(-\frac{3}{8}\,a\right)(0)\right] \end{aligned}$$

$$\begin{aligned} I_{xy34} &= I_{x_c\,y_{c34}} + A_{34}\,[dx_{34}\,dy_{34}] \\ &= 0 + (a)(2t)\left[\left(\frac{1}{8}\,a\right)(a)\right] \end{aligned}$$

$$\begin{aligned} I_{xy} &= I_{xy12} + I_{xy23} + I_{xy34} \\ &= \frac{2}{8}\,t\,a^3 + 0 - \frac{10}{8}\,t\,a^3 = -t\,a^3 \end{aligned}$$

Second moments of area about the centroid

$$I_{xx} = \frac{16}{3}\,t\,a^3 \qquad I_{yy} = \frac{53}{24}\,t\,a^3 \qquad I_{xy} = -t\,a^3 \tag{5}$$

The factor in the denominator is

$$I_{xx}\,I_{yy} - I_{xy}^2 = \frac{97}{9}\,a^6\,t^2 \tag{6}$$

Ans.