AOE 3024: Thin Walled Structures
Solutions to Homework # 11

For the thin-walled section shown, determine the shear flow in all the panels and determine the location of the shear center.

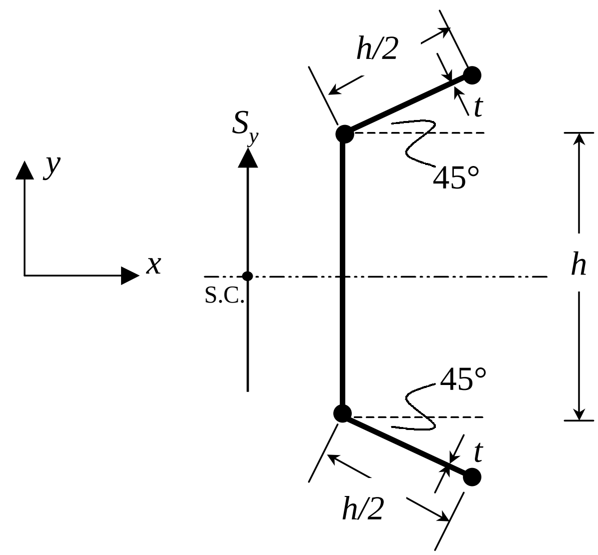

Fig. 1 Symmetrically thin-walled channel section.

FIRST: Understand the problem

Since this is a symmetrical thin-walled section, we do not need to calculate I_{xy} because it is zero. Therefore, only I_{xx} and I_{yy} needs to be calculated. The location of the centroid will be needed. By inspection $y_c = 0$ and x_c needs to be calculated.

Next, before we proceed let us decide where to take the torque equivalence. Recall that the torque equivalence about a point, say O_s, determines the location of the shear center. Further, arbitrarily apply shear loads S_y and S_x at the shear center S. Note because of symmetry $S_x = 0$.

Torque equivalence can be taken about any point. However, for some points less calculation will be needed. For instance, if we take moment about the shear center then the shear flow acting at each branch will have to be calculated. If we choose point 1 or 4, then the shear flow acting in two branches will have to be calculated.

For point 2 or 3, only the shear flow along one of the branches needs to be calculated. If torque equivalence is taken about a point 3, then only calculate $q_{12}(s)$. If torque equivalence is taken about a point 2, then only calculate $q_{43}(s)$.

Regardless about what point we take the torque equivalence, we should get the same answer. Let us choose point 2 for these solutions.

If point $O_s = 2$, then we should consider Fig. 2

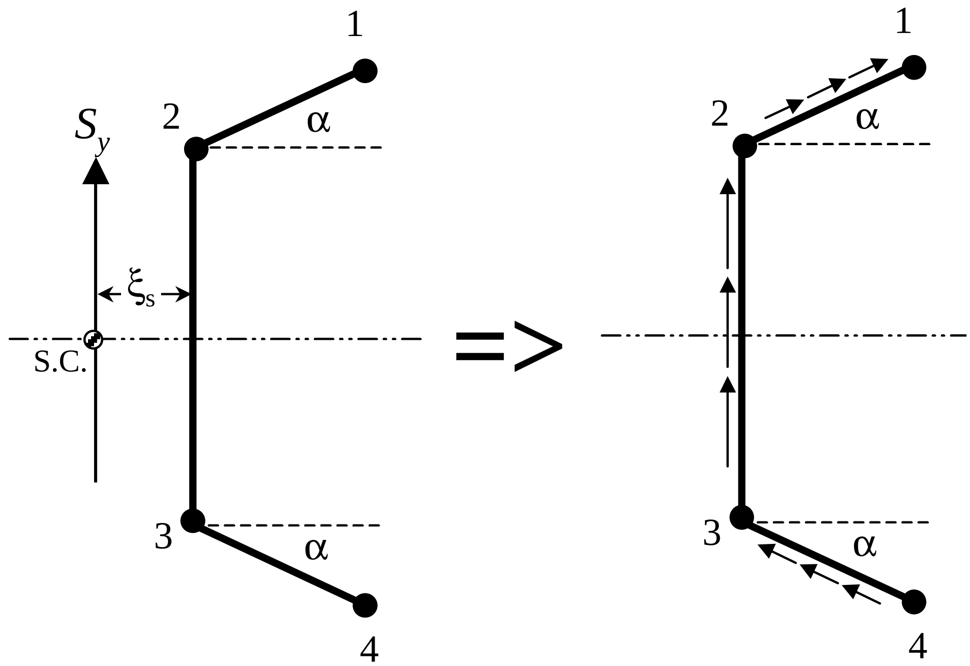

Fig. 2 Shear flow convention

BASIC EQUATIONS:

Shear flow distribution

When S_y is applied through the shear center then no torsion exists and the shear flow distribution is given by ($S_x = 0$ and $I_{xy} = 0$)

$$q_i(s) = q_{o_i} - \left(\frac{S_y}{I_{xx}}\right) \int_0^s t_i(s_i)\, y_i(s_i)\, ds_i \tag{1}$$

where q_{o_i} is found by evaluating $q_i(s = 0)$. When the thickness is constant along each branch: $t_i(s_i) = t_i$.

Shear flow across the stiffener,

$$\triangle q_s = -\frac{S_y}{I_{xx}}\, B_s\, y_s \tag{2}$$

Torque equivalence

Torque equivalence about point 2, determines the location ξ_s of the shear center. For a positive torque counterclockwise (see Fig. 2),
for $S_x = 0$ and $S_y > 0$

$$-S_y\,\xi_s = \sum_i \int_0^{a_i} \overrightarrow{r}_i(s) \times \overrightarrow{q}_i(s)\,ds \tag{3}$$

where i represents the i^{th} branch, a_i the length of the branch, and $r_i(s)$ the distance perpendicular from the point 2 to the contour s_i.

For the present solution we will take torque equivalence about point 2 and use Fig. 2. In solving this problem, consider a three-step solution procedure:

1. Calculate the centroid: x_c
2. Determine the second moments of area about the centroid: I_{xx}
3. Find the shear flow distribution in each branch, $q_i(s)$, to find the horizontal location, ξ_s, of the shear center by considering $S_x = 0$ and $S_y > 0$

I. Centroid

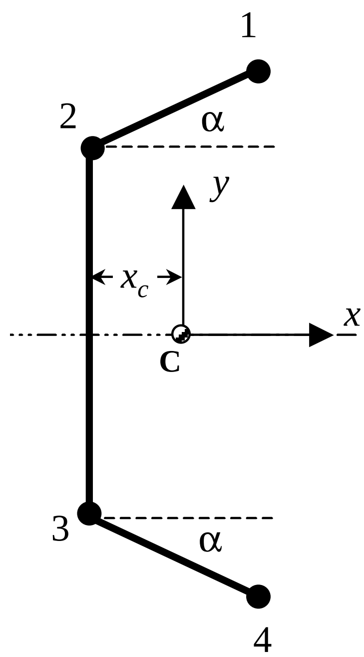

Fig. 3 Calculation of the centroid.

Let's place the origin at the middle of the web (23) and calculate x_c and y_c.

BRANCHES:

Section	x_i	y_i	A_i
12	$\frac{h}{4}\cos\alpha$	$\frac{h}{2}+\frac{h}{4}\sin\alpha$	$\frac{ht}{2}$
23	0	0	ht
34	$\frac{h}{4}\cos\alpha$	$-\frac{h}{2}-\frac{h}{4}\sin\alpha$	$\frac{ht}{2}$

STRINGERS:

Section	xs_i	ys_i	B_{s_i}
1	$\frac{h}{2}\cos\alpha$	$\frac{h}{2}+\frac{h}{2}\sin\alpha$	$\frac{ht}{2}$
2	0	$\frac{h}{2}$	$\frac{ht}{2}$
3	0	$-\frac{h}{2}$	$\frac{ht}{2}$
4	$\frac{h}{2}\cos\alpha$	$-\frac{h}{2}-\frac{h}{2}\sin\alpha$	$\frac{ht}{2}$

$$A_T = \sum A_i + \sum B_{s_i} = 4\,h\,t$$

$$Q_x = \sum x_i\,A_i + \sum xs_i\,B_{s_i} = \frac{3}{4}h^2\,t\,\cos\alpha \qquad Q_y = \sum y_i\,A_i + \sum ys_i\,B_{s_i} = 0$$

$$x_c = \frac{Q_x}{A_T} = \frac{3}{16}h\,\cos\alpha \qquad y_c = \frac{Q_y}{A_T} = 0$$

$$\boxed{x_c = \frac{3}{16}h\,\cos\alpha \qquad y_c = 0} \tag{4}$$

Ans.

II. Second moments of area

Using thin-walled assumption

BRANCHES:

Section	I_{xc_i}	I_{yc_i}	$I_{xc\,yc_i}$
12	$(t\,h^3)/12$	0	0
23	$\left(t\,(h/2)^3\right)/12\,\sin^2\alpha$	$\left(t\,(h/2)^3\right)/12\,\cos^2\alpha$	0
34	$\left(t\,(h/2)^3\right)/12\,\sin^2\alpha$	$\left(t\,(h/2)^3\right)/12\,\cos^2\alpha$	0

The stringers are assumed not having any second moment of area. From symmetry $I_{xy} = 0$. Second Moment of area I_{xx} is

$$
\begin{aligned}
I_{xx12} &= I_{xc12} + A_{12}\,[0]^2 \\
I_{xx23} &= I_{xc23} + A_{23}\left[\frac{h}{2} + \frac{h}{4}\sin\alpha\right]^2 \\
I_{xx34} &= I_{xc34} + A_{34}\left[-\frac{h}{2} - \frac{h}{4}\sin\alpha\right]^2 \\
\\
Is_{xx_1} &= B_{s_1}\left[\frac{h}{2} + \frac{h}{2}\sin\alpha\right]^2 \\
Is_{xx_2} &= B_{s_2}\left[\frac{h}{2}\right]^2 \\
Is_{xx_3} &= B_{s_3}\left[-\frac{h}{2}\right]^2 \\
Is_{xx_4} &= B_{s_4}\left[-\frac{h}{2} - \frac{h}{2}\sin\alpha\right]^2 \\
\\
I_{xx} &= I_{xx12} + I_{xx23} + I_{xx34} + Is_{xx_1} + Is_{xx_2} + Is_{xx_3} + Is_{xx_4} \\
&= \frac{5\,h^3\,t}{6} + \frac{3\,h^3\,t\,\sin\alpha}{4} + \frac{h^3\,t\,\sin^2\alpha}{3} \\
&= h^3\,t + \frac{3\,h^3\,t}{4\sqrt{2}} = 1.53033\,h^3\,t
\end{aligned}
$$

Second Moment of area I_{yy}

$$
\begin{aligned}
I_{yy_{12}} &= I_{y_{c12}} + A_{12}\left[-x_c\right]^2 \\
I_{yy_{23}} &= I_{y_{c23}} + A_{23}\left[\frac{h}{4}\cos\alpha - x_c\right]^2 \\
I_{yy_{34}} &= I_{y_{c34}} + A_{34}\left[\frac{h}{4}\cos\alpha - x_c\right]^2
\end{aligned}
$$

$$
\begin{aligned}
Is_{yy_1} &= B_{s_1}\left[\frac{h}{2}\cos\alpha - x_c\right]^2 \\
Is_{yy_2} &= B_{s_2}\left[-x_c\right]^2 \\
Is_{yy_3} &= B_{s_3}\left[-x_c\right]^2 \\
Is_{yy_4} &= B_{s_4}\left[\frac{h}{2}\cos\alpha - x_c\right]^2
\end{aligned}
$$

$$
\begin{aligned}
I_{yy} &= I_{yy_{12}} + I_{yy_{23}} + I_{yy_{34}} + Is_{yy_1} + Is_{yy_2} + Is_{yy_3} + Is_{yy_4} \\
&= \frac{37}{192}\cos^2\alpha \\
&= \frac{37}{384} = 0.0963542\,h^3\,t
\end{aligned}
$$

Second moments of area about the centroid

$$
I_{xx} = 1.53033\,h^3\,t \tag{5}
$$

Ans.

III. Shear Flow and shear center

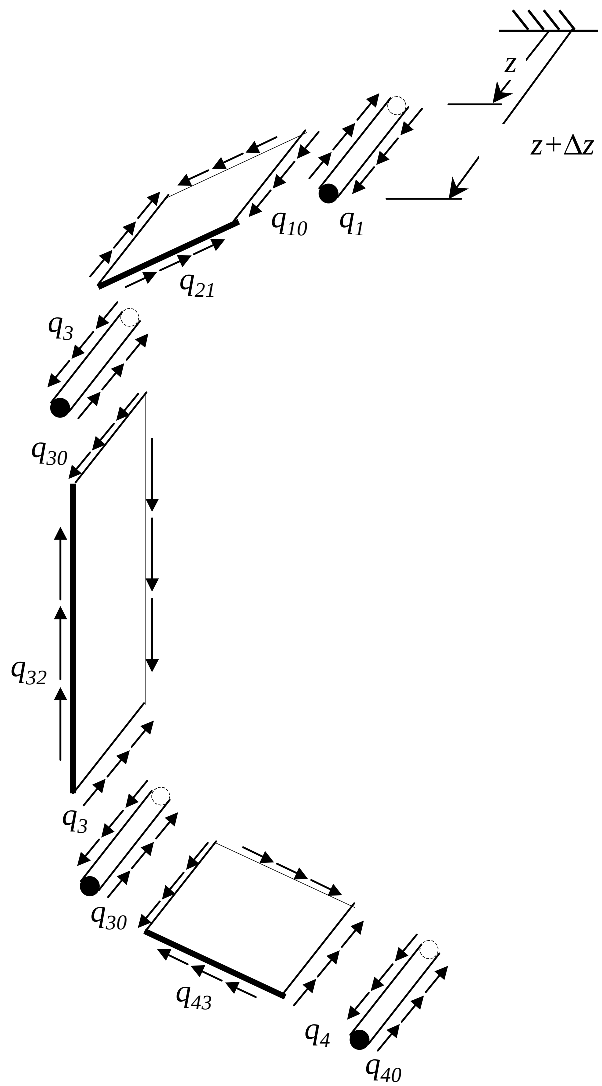

Fig. 4 Free body diagram of each section. These are consistent with the assumed shear flow convention.

In order to calculate the shear flow in each flange, we need to first find the $y_i(s_i)$ for each part.

Following the assumed flow convention in Fig. 4 and from the geometry of the cross-section we get: (about the centroid)

For flange 21,

$$a_{21} = h/2 \qquad s_{21} = 0 \qquad \Rightarrow \qquad y_{21} = \frac{h}{2} \tag{6}$$

$$s_{21} = \frac{h}{2} \qquad \Rightarrow \qquad y_{21} = \frac{h}{2} + \frac{h}{2}\sin\alpha \tag{7}$$

For flange 32,

$$a_{32} = h \qquad s_{32} = 0 \quad \Rightarrow \quad y_{32} = -\frac{h}{2} \tag{8}$$

$$s_{32} = h \quad \Rightarrow \quad y_{32} = \frac{h}{2} \tag{9}$$

For flange 43,

$$a_{43} = h/2 \qquad s_{43} = 0 \quad \Rightarrow \quad y_{43} = -\frac{h}{2} - \frac{h}{2}\sin\alpha \tag{10}$$

$$s_{43} = \frac{h}{2} \quad \Rightarrow \quad y_{43} = -\frac{h}{2} \tag{11}$$

Since these are straight lines, the equation of the line to can be used to calculate $y_i(s_i)$,

$$y_i(s_i) = \frac{y_i\Big|_{(s_i=a_i)} - y_i\Big|_{(s_i=0)}}{a_i - 0}\, s_i + b \qquad \text{where} \qquad b = y_i\Big|_{(s_i=0)} \tag{12}$$

where a_i is the length of the i^{th} branch.

Across stringer 4, ($q_{o_4} = 0$, because it is a free edge)

$$B_{s_4} = \frac{ht} {2} \qquad y_{s_4} = -\frac{h}{2} - \frac{h}{2}\sin\alpha$$

$$q_4 = q_{o_4} - \frac{S_y}{I_{xx}}\, B_{s_4}\, y_{s_4} = \frac{\left(5+\sqrt{2}\right)\, S_y}{23\, h} \tag{13}$$

For flange 43, ($t_{43} = t$)

$$y_{43}(s_{43}) = -\frac{h}{2} - \frac{h}{2\sqrt{2}} + \frac{s_{43}}{\sqrt{2}}$$

$$q_{43}(s) = q_{o_{43}} - \frac{S_y}{I_{xx}} \int_0^s t_{43}\, y_{43}(s_{43})\, ds_{43}$$

$$q_{o_{43}} = q_{43}(0) = q_4 = \frac{\left(5+\sqrt{2}\right)\, S_y}{23\, h}$$

$$q_{43}(s) = \frac{2\,h^2\,S_y}{8\,h^3 + 3\sqrt{2}\,h^3} + \frac{\sqrt{2}\,h^2\,S_y}{8\,h^3 + 3\sqrt{2}\,h^3} + \frac{4\,h\,s\,S_y}{8\,h^3 + 3\sqrt{2}\,h^3} + \frac{2\sqrt{2}\,h\,s\,S_y}{8\,h^3 + 3\sqrt{2}\,h^3} - \frac{2\sqrt{2}\,s^2\,S_y}{8\,h^3 + 3\sqrt{2}\,h^3} \tag{14}$$

Across stringer 3,

$$\begin{aligned} B_{s_3} &= \frac{ht}{2} \qquad y_{s_3} = -\frac{h}{2} \\ q_3 &= q_{o_3} - \frac{S_y}{I_{xx}} B_{s_3}\, y_{s_3} \\ q_{o_3} &= q_{43}(\frac{h}{2}) = \frac{S_y}{2\,h} \\ q_3 &= \frac{3\,\left(13 - 2\,\sqrt{2}\right)\,S_y}{46\,h} \end{aligned} \tag{15}$$

For flange 32, $(t_{32} = t)$

$$\begin{aligned} y_{32}(s_{32}) &= \frac{-h}{2} + s_{32} \\ q_{32}(s) &= q_{o_{32}} - \frac{S_y}{I_{xx}} \int_0^s t_{32}\, y_{32}(s_{32})\, ds_{32} \\ q_{o_{32}} &= q_{32}(0) = q_3 = \frac{3\,\left(13 - 2\,\sqrt{2}\right)\,S_y}{46\,h} \\ q_{32}(s) &= \frac{12\,h^2\,S_y}{16\,h^3 + 6\,\sqrt{2}\,h^3} + \frac{3\,\sqrt{2}\,h^2\,S_y}{16\,h^3 + 6\,\sqrt{2}\,h^3} \\ &\quad + \frac{8\,h\,s\,S_y}{16\,h^3 + 6\,\sqrt{2}\,h^3} - \frac{8\,s^2\,S_y}{16\,h^3 + 6\,\sqrt{2}\,h^3} \end{aligned} \tag{16}$$

Across stringer 2,

$$\begin{aligned} B_{s_2} &= \frac{ht}{2} \qquad y_{s_2} = \frac{h}{2} \\ q_2 &= q_{o_2} - \frac{S_y}{I_{xx}} B_{s_2}\, y_{s_2} \\ q_{o_2} &= q_{32}(h) = \frac{3\,\left(13 - 2\,\sqrt{2}\right)\,S_y}{46\,h} \\ q_2 &= \frac{S_y}{2\,h} \end{aligned} \tag{17}$$

For flange 21, $(t_{21} = t)$

$$y_{21}(s_{21}) = \frac{h}{2} + \frac{s_{21}}{\sqrt{2}}$$

$$q_{21}(s) = q_{o_{21}} - \frac{S_y}{I_{xx}} \int_0^s t_{21}\, y_{21}(s_{21})\, ds_{21}$$

$$q_{o_{21}} = q_{21}(0) = q_2 \frac{S_y}{2\,h}$$

$$q_{21}(s) = \frac{8\,h^2\,S_y}{16\,h^3 + 6\,\sqrt{2}\,h^3} + \frac{3\,\sqrt{2}\,h^2\,S_y}{16\,h^3 + 6\,\sqrt{2}\,h^3} - \frac{8\,h\,s\,S_y}{16\,h^3 + 6\,\sqrt{2}\,h^3} - \frac{4\,\sqrt{2}\,s^2\,S_y}{16\,h^3 + 6\,\sqrt{2}\,h^3} \tag{18}$$

Check: at stringer 1 the static equilibrium must be satisfied,

$$q_1 = \text{must be zero}$$

$$B_{s_1} = \frac{ht}{2} \qquad y_{s_1} = \frac{h}{2} + \frac{h}{2}\,\sin\alpha$$

$$q_1 = q_{o_1} - \frac{S_y}{I_{xx}}\,B_{s_2}\,y_{s_2}$$

$$q_{o_1} = q_{21}(\frac{h}{2}) = \frac{\left(5 + \sqrt{2}\right)\,S_y}{23\,h}$$

$$q_1 = 0 \text{ ...GOOD!} \tag{19}$$

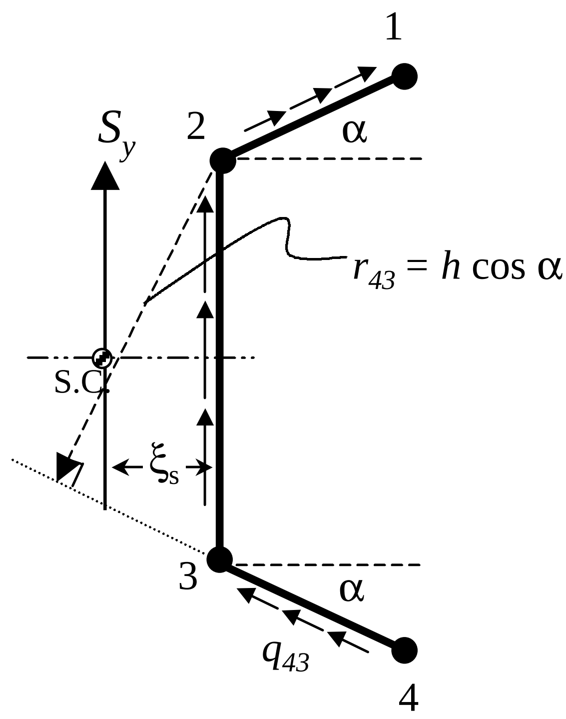

Fig. 5

The shear center is found as follows

$$-S_y\,\xi_s \quad = \quad -\int_0^{a_{43}} r_{43}(s)\,q_{43}(s)\,ds \tag{20}$$

where $r_{43}(s)$ is shown in Fig. 5.

Therefore,

$$\begin{aligned} -S_y\,\xi_s &= -\int_0^{a_{43}} r_{43}(s)\,q_{43}(s)\,ds \\ &= -\int_0^{\frac{h}{2}} (h\,\cos\alpha)\;q_{43}(s)\,ds \end{aligned} \tag{21}$$

Substituting $q_{43}(s)$ from Eq. (14) and integrating

$$= -\frac{\left(5+24\sqrt{2}\right)\,h\,S_y}{276}$$

Multiplying by -1

$$S_y\,\xi_s = \frac{\left(5+24\sqrt{2}\right)\,h\,S_y}{276} \tag{22}$$

$$\boxed{\xi_s = \frac{\left(5+24\sqrt{2}\right)\,h}{276} = 0.141\,h}$$ Ans.