AOE 3024: Thin Walled Structures
Homework # 3: Due Wednesday, September 19, 2001

NAME PLEDGE

a. Determine whether the following state of plane stress within an elastic solid is admissible. Note that A is constant. (10pts)

$$\sigma_{xx} = 2\,A\,x^2 \quad (1a)$$
$$\sigma_{yy} = 2\,A\,\left(4\,x^2 + y^2\right) \quad (1b)$$
$$\tau_{xy} = -4\,A\,x\,y \quad (1c)$$
$$\tau_{yx} = \tau_{xy} \quad (1d)$$
$$\tau_{xz} = \tau_{yz} = \tau_{zx} = \tau_{zy} = \sigma_{zz} = 0 \quad (1e)$$

Any state of stress satisfying the equilibrium equations is statically admissible. Assuming that body forces are negligible, the equilibrium equations can be written as follows

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \quad (2a)$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0 \quad (2b)$$
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0 \quad (2c)$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \frac{\partial (2\,A\,x^2)}{\partial x} + \frac{\partial (-4\,A\,x\,y)}{\partial y} + \frac{\partial (0)}{\partial z} \quad (3a)$$
$$= 4\,A\,x - 4\,A\,x + 0 \quad (3b)$$
$$= 0 \qquad \text{satisfies first equilibrium equation} \quad (3c)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = \frac{\partial (-4\,A\,x\,y)}{\partial x} + \frac{\partial (8\,A\,x^2 + 2\,A\,y^2)}{\partial y} + \frac{\partial (0)}{\partial z} \quad (4a)$$
$$= -4\,A\,y + 4\,A\,y + 0 \quad (4b)$$
$$= 0 \qquad \text{satisfies second equilibrium equation} \quad (4c)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \frac{\partial (0)}{\partial x} + \frac{\partial (0)}{\partial y} + \frac{\partial (0)}{\partial z} \quad (5a)$$
$$= 0 + 0 + 0 \quad (5b)$$
$$= 0 \qquad \text{satisfies third equilibrium equation} \quad (5c)$$

Since the state of stress satisfies all three equilibrium equations and also $\tau_{xy} = \tau_{yx}$ (for moment equilibrium) the given stress state is a statically admissible one.

b. An element in plane stress at the surface of a wing panel is subjected to the following stresses

$$[\boldsymbol{\sigma}] = \begin{bmatrix} 15 & -4 & 0 \\ -4 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa} \tag{6}$$

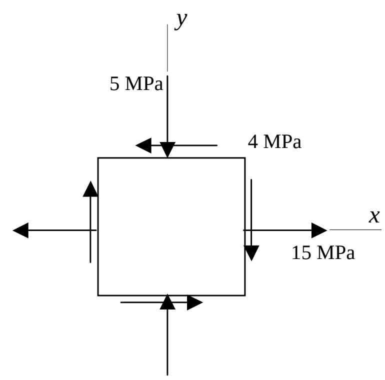


Fig. 1

Considering only the in-plane stresses and using Mohr Circle determine:

1. **Stresses acting on a element inclined at an angle $\theta = 40°$. (10pts)**
2. **Principal stresses and maximum shear stresses. (10pts)**

(Show all results on sketches of properly oriented elements)

Step I. Calculate the radius and Mohr's circle center

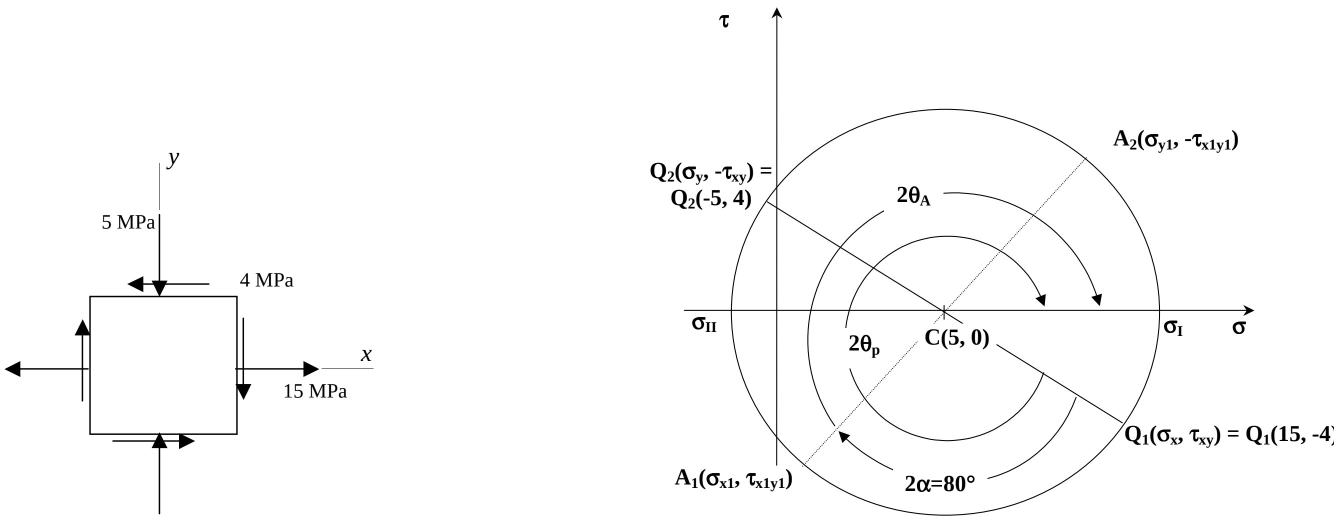
$$\sigma_{ave} = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \frac{(15) + (-5)}{2} \text{ MPa} = 5 \text{ MPa} \tag{7}$$

$$\sigma_{dif} = \frac{\sigma_{xx} - \sigma_{yy}}{2} = \frac{(15) - (-5)}{2} \text{ MPa} = 10 \text{ MPa} \tag{8}$$

$$R = \sqrt{\tau_{xy}^2 + \sigma_{dif}^2} = \sqrt{(-4)^2 + (10)^2} \text{ MPa} = 10.7703 \text{ MPa} \tag{9}$$

$$C = C(\sigma_{ave}, 0) = C(5 \text{ MPa}, 0) \tag{10}$$

Step II. Draw the circle and locate all points



a) stresses on a two dimensional element

b) Mohr's circle for plane stress in the xy plane

Fig. 2

Step III. Calculate angles: (All measured positive clockwise)

Recall that all angles are measured positive clockwise in the Mohr's circle but are positive counterclockwise in the rotation of the differential element. Therefore, the negative sign is an indication that the rotation in the Mohr's circle is counterclockwise but clockwise in the differential element.

Principal stresses act on an element inclined at an angle θ_p

$$2\theta_p' = \tan^{-1}\left[\frac{\tau_{xy}}{\sigma_{dif}}\right] = \tan^{-1}\left[\frac{(-4)}{(10)}\right] = -21.8014° \tag{11a}$$

$$2\theta_p = 2\theta_p' + 360° = 338.199° \tag{11b}$$

$$\theta_p = 169.099° \tag{11c}$$

Since $2\,\theta_p$ is measured from $\overline{Q_1 C}$ to positive σ-axis, we add 360° to measure the angle clockwise. Maximum shear stresses act on an element inclined at an angle θ_s

$$2\theta_s = 2\theta_p \pm 90° = 338.199° \pm 90° \tag{12a}$$

$$\theta_s = \theta_p \pm 45° = 169.099° \pm 45° \tag{12b}$$

Transformed stresses act on an element inclined at an angle $\alpha = 40°$

$$2\,\theta_A = 2\,\theta_p - 2\,\alpha = 338.199° - 80° = 258.199° \tag{13a}$$

Step IV. Determine the normal stresses

The normal stresses acting on an element inclined at an angle α are

$$\sigma_{x_1} = \sigma_{ave} + R\,\cos(2\,\theta_A) = (5) + (10.7703)\,\cos(258.199°) = 2.79725 \text{ MPa} \tag{14}$$

$$\sigma_{y_1} = \sigma_{ave} - R\,\cos(2\,\theta_A) = (5) - (10.7703)\,\cos(258.199°) = 7.20275 \text{ MPa} \tag{15}$$

Note that when calculating principal stresses $2\,\alpha = 2\,\theta_p \rightarrow 2\,\theta_A = 0°$, therefore the principal stresses are

$$\sigma_I \quad = \quad \sigma_{ave} + R = (5) + (10.7703) = 15.7703 \text{ MPa} \tag{16}$$

$$\sigma_{II} \quad = \quad \sigma_{ave} - R = (5) - (10.7703) = -5.77033 \text{ MPa} \tag{17}$$

Step V. Determine the shear stress

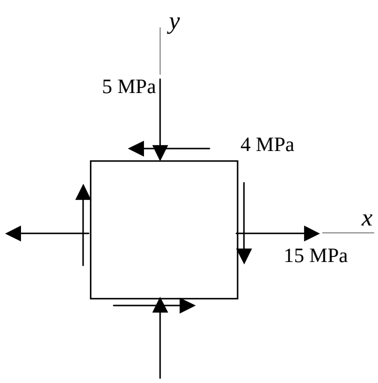
The shear stresses acting on an element inclined at an angle α are

$$\tau_{x_1 y_1} \quad = \quad R\,\sin\left(2\,\theta_A\right) = (10.7703)\,\sin\left(258.199°\right) = -10.5427 \text{ MPa} \tag{18}$$

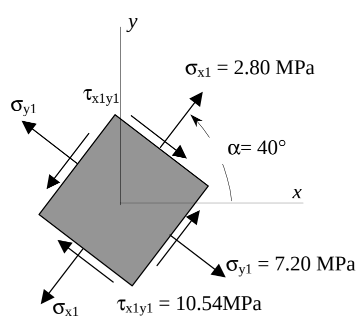
The maximum shear stresses acting on an element inclined at an angle θ_s are

$$\tau_{max}\Big|_{in-plane} \quad = \quad R = \frac{\sigma_I - \sigma_{II}}{2} = 10.7703 \text{ MPa} \tag{19}$$

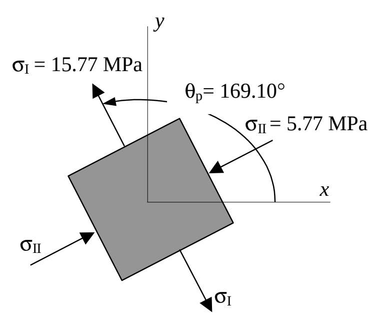
Step VI. Show all results on sketches of properly oriented elements



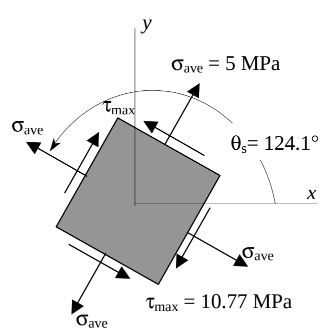
a) Stresses acting on an element in plane stress



b) Stresses acting on an element oriented at an angle $\theta = \alpha$



c) Principal normal stresses



d) Maximum shear stresses

Fig. 3